JHEP HYPER VERSION

RECEIVED: *May 8, 2001*, ACCEPTED: *July 27, 2001*

$D = 10$ super–Yang-Mills at $O(\alpha'^2)$

Martin Cederwall, Bengt E.W. Nilsson and Dimitrios Tsimpis

Department of Theoretical Physics
Göteborg University and Chalmers University of Technology
SE-412 96 Göteborg, Sweden
E-mail: martin.cederwall@fy.chalmers.se bengt.nilsson@fy.chalmers.se tsimpis@fy.chalmers.se

ABSTRACT: Using superspace techniques, the complete and most general action of $D = 10$ super–Yang-Mills theory is constructed at the α'^2 level. No other approximations, e.g., keeping only a subset of the allowed derivative terms, are used. The Lorentz structure of the α'^2 corrections is completely determined, while (depending on the gauge group) there is some freedom in the adjoint structure, which is given by a totally symmetric four-index tensor. We examine the second, non-linearly realised supersymmetry that may be present when the gauge group has a U(1) factor, and find that the constraints from linear and non-linear supersymmetry to a large extent coincide. However, the additional restrictions on the adjoint structure of the order α'^2 interactions following from the requirement of non-linear supersymmetry do not completely specify the symmetrised trace prescription.

KEYWORDS: Field Theories in Higher Dimensions, Gauge Symmetry, Supersymmetric Effective Theories, Superspaces.

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Contents

1. Introduction

One of the most fascinating results in connection with open strings and D-branes is the discovery [1, 2] that for an abelian and constant field strength the α' expansion at string tree level can be summed up to yield the Born-Infeld theory [3]. The appearence of the Born-Infeld lagrangian can also be understood in other ways, e.g., through the superembedding formalism [4] or as a direct consequence of requiring the existence of deformed BPS conditions [5]. This latter approach probably leads to very similar if not identical results as demanding linear supersymmetry in ten dimensions. However, if one relaxes the condition of constant field strength the theory becomes substantially more complicated. A closed form for the lagrangian is not known in this case and the action has to be obtained order by order in the number of derivatives correcting the original Born-Infeld action [6, 2, 7, 8]. For a review of these and other results regarding the role of the Born-Infeld theory in string theory, see ref. [9]. Further interesting results in this direction using open-string-related quantities [10] can be found in refs. [11, 12].

The fact that non-abelian gauge theories are expected to describe a stack of coincident D-branes [13] raises many questions concerning the non-abelian generalisation of the abelian results alluded to above. As noticed by Tseytlin [14] a large class of the derivative terms is accounted for by using a non-abelian Born-Infeld action defined in terms of a symmetrised trace, STr, over the adjoint gauge group indices.

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In the non-abelian case, however, there are to our knowledge no results indicating that the action of the full theory, without any approximations, should be expressible to all orders in α' in closed form similar to the abelian Born-Infeld theory. Therefore it is of some interest to derive the full non-abelian D-brane action order by order in the α' expansion. Doing this directly from string theory quickly becomes rather difficult and it would be useful to find other means of obtaining these results. There are several ideas on the market, most of which are in one way or another related to supersymmetry. In the abelian case, many systems with Born-Infeld type actions and less than maximal linear supersymmetry have also a non-linear supersymmetry that can be seen to follow by a Goldstone mechanism [15] from a theory in which all supersymmetries are linearly realised. Also the superembedding formalism [4] is known to give rise to non-linear supersymmetries on the branes although in a much more indirect way [16], but, on the other hand, in this approach κ-symmetry arises very naturally [4]. For maximally supersymmetric D-branes, their κ-symmetric actions [17] can easily be gauge-fixed and seen to reduce to systems with maximal linear supersymmetry [18] in less than ten dimensions. The Born-Infeld type actions obtained in this way can be immediately generalised to the ten-dimensional abelian vector multiplet. In cases with maximal supersymmetry, however, non-linear supersymmetries are harder to realise and are at this point very poorly understood.

In the non-abelian situation it is known that linear supersymmetry fixes uniquely the action not only at lowest order but also at order α'^2 at least if one starts from $\mathrm{STr}\,F^4$ as done in ref. [19]. The trace STr refers to the symmetrised trace in the fundamental representation introduced in [6]. Later it was proposed by Tseytlin [14] that the non-abelian Born-Infeld action might be of the same form as the abelian one, which is the case if a symmetric ordering prescription is imposed by means of the symmetric trace. Later work has indicated that there are deviations from the STr prescription [20, 21], but the situation is still rather unclear.

One may also try to deduce the form of the non-abelian action by means of non-abelian generalisations of other symmetries appearing in the abelian case, e.g., the non-linear supersymmetry or the κ symmetry. Trying to use parameters valued in the adjoint of the gauge group seems rather involved and would probably, if it could be realised, have very interesting implications in connection with non-commutative space-times and matrix valued coordinates [22] (see also the discussion in ref. [23]). An attempt to implement such ideas in the case of κ-symmetry is described in ref. [24]. A different approach, not directly related to supersymmetry, to deduce the structure of non-abelian Born-Infeld theory is used in refs. [25, 26]. In these papers the authors exploit instead the background invariance related to the Seiberg-Witten map [10].

In this paper we continue our investigations of the ten-dimensional non-abelian supersymmetric Yang-Mills theory that we initiated in ref. [27]. Our approach relies on implementing only the linear supersymmetry in ten dimensions which is done in a manifest fashion through the use of superspace (see, e.g., ref. [28]). By solving the Bianchi identities (BI) for the non-abelian superfield strength F_{AB}[1] imposing no constraints except for conventional ones [29],[2] we managed in ref. [27] to solve the BI's and rewrite them as a set of algebraic relations between the ordinary physical fields and a set of pseudo-auxiliary fields (here the prefix 'pseudo' is used to emphasise the fact that these auxiliary fields are not sufficient to construct an action). Some of these relations involve derivatives on the physical fields which indicate that they will become field equations as soon as the superfield strength component of lowest dimension, $F_{\alpha\beta}$, is given explicitly in terms of the physical fields. When we feed such an explicit form of $F_{\alpha\beta}$ through the solution to the BI's the corresponding form of the field equations is obtained.

Not knowing the exact form of $F_{\alpha\beta}$, this program has to be carried out in an iterative fashion order by order in α'. In order to find the most general form of the action compatible with linear supersymmetry we thus have to construct all possible expressions at each order in the α' expansion of $F_{\alpha\beta}$. At order α'^2 this was done in ref. [27] proving that there is only one relevant expression in

$$F_{\alpha\beta} = \frac{1}{5!}\Gamma_{\alpha\beta}^{a_1\ldots a_5} J_{a_1\ldots a_5} \tag{1.1}$$

(the vector term is set to zero as a conventional constraint), namely[3]

$$J^A_{abcde} = -\frac{1}{2}\alpha'^2 M^A{}_{BCD}(\lambda^B\Gamma^f\Gamma_{abcde}\Gamma^g\lambda^C)F^D_{fg}\,. \tag{1.2}$$

Here $M^A{}_{BCD}$ is totally symmetric in all four indices and must be constructed out of the invariant tensors of the gauge group. As we will see in section 4, this is a less restricted form of the (unique) symmetrised trace, STr, appearing in string theory [6] at order α'^2. The above form of $F_{\alpha\beta}$ has appeared in the literature before [30, 19] but without proof of its unique status at this order. In the latter of these references the argument was turned around in the sense that the component action was constructed first by means of Noether methods, and the result was subsequently used to derive the required form of $F_{\alpha\beta}$. However, the symmetrised trace was used as input in the first step and the derivation of (1.2) was therefore less general than the one presented in ref. [27]. The resulting component action is derived in section 2, and presented here with explicit adjoint group indices $A, B, C, \ldots$:

$$\begin{aligned}\mathcal{L}' &= \mathcal{L}'^{(0)} + \alpha'^2\mathcal{L}'^{(2)}\\ &= -\frac{1}{4}G^{Aij}G^A_{ij} + \frac{1}{2}\chi^A \not{D}\chi^A -\\ &\quad -6\alpha'^2 M_{ABCD}\Big[\operatorname{tr} G^A G^B G^C G^D - \frac{1}{4}(\operatorname{tr} G^A G^B)(\operatorname{tr} G^C G^D) -\end{aligned}$$

[1] Here $A,B,\ldots$ refer to the combined (vector, spinor) indices $(a,\alpha),(b,\beta),\ldots$, see ref. [27] for conventions.
[2] This was also suggested in ref. [30].
[3] The overall sign is changed as compared to the expression given in ref. [27].

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$$- 2G^{A\,i}{}_k G^{B\,jk}(\chi^C\Gamma_i D_j\chi^D) + \frac{1}{2}G^{A\,il}D_l G^{B\,jk}(\chi^C\Gamma_{ijk}\chi^D) +$$
$$+ \frac{1}{180}(\chi^A\Gamma^{ijk}\chi^B)(D_l\chi^C\Gamma_{ijk}D^l\chi^D) +$$
$$+ \frac{3}{10}(\chi^A\Gamma^{ijk}\chi^B)(D_i\chi^C\Gamma_j D_k\chi^D) +$$
$$+ \frac{7}{60}f^D{}_{EF}G^{A\,ij}(\chi^B\Gamma_{ijk}\chi^C)(\chi^E\Gamma^k\chi^F) -$$
$$- \frac{1}{360}f^D{}_{EF}G^{A\,ij}(\chi^B\Gamma^{klm}\chi^C)(\chi^E\Gamma_{ijklm}\chi^F)\Big] + O(\alpha'^3)\,. \quad (1.3)$$

The trace tr refers to the structure of the Lorentz indices. Note also that it is only in the last two terms that the non-abelian structure is crucial since $M^A{}_{BCD}$ is non-zero also in the abelian case. As discussed in section 5, in the non-abelian case the number of independent structures in $M^A{}_{BCD}$ depends on the gauge group.

This paper is organised as follows. In section 2 we insert eq. (1.2) into the equations obtained in ref. [27] from the BI's. The result is the equations of motion, which in section 3 are integrated to the complete action at order α'^2 including all fermionic terms required by supersymmetry at this order. The answer agrees exactly with previously derived terms in, e.g., ref. [19] where however the quartic fermion terms were not given. In section 4 we present the form of the supersymmetry transformations to the order relevant for our purposes. We also show that there exists an α'^2-corrected non-linear supersymmetry provided the gauge group has a U(1) factor and the parameter takes values only in this factor. The corrections at the level considered are exactly the ones responsible for turning this symmetry from an abelian one into an ordinary second supersymmetry. Interestingly enough, we find that also the fields in the semi-simple part of the gauge group transform under this non-linear supersymmetry. However, since the parameter is abelian this symmetry does not have any implications for questions concerning non-commutative space-times. section 5 is devoted to an analysis of the possible ways of constructing $M^A{}_{BCD}$, which turn out to be rather mildly affected by requiring the existence of a non-linear supersymmetry. We collect our results and some additional comments in section 6.

2. Interactions at $O(\alpha'^2)$ from superspace

In order to derive the field equations for A_a and λ^α we must insert (suppressing both α' and $M^A{}_{BCD}$ in the following)

$$J_{abcde} = 10(\lambda\Gamma_{[abc}\lambda)F_{de]} + \frac{1}{2}(\lambda\Gamma_{abcde}{}^{fg}\lambda)F_{fg} = -\frac{1}{2}(\lambda\Gamma^f\Gamma_{abcde}\Gamma^g\lambda)F_{fg} \qquad (2.1)$$

into the following exact expressions obtained in ref. [27] by solving the superspace BI's:

$$0 = D^bF_{ab} - \lambda\Gamma_a\lambda - 8D^bK_{ab} + 36w_a - \frac{4}{3}\{\lambda, \tilde{J}_a\} - 2\tilde{J}_b\Gamma_a\tilde{J}^b +$$

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$$+\frac{1}{140\cdot 3!}\tilde{J}_{bcd}\Gamma_a\tilde{J}^{bcd}+\frac{1}{42}[K_{bcde},J_a{}^{bcde}]+\frac{1}{42\cdot 4!}[D^fJ_{fbcde},J_a{}^{bcde}] \qquad (2.2)$$

and

$$0=\not{D}\lambda-30\psi+\frac{4}{3}D^a\tilde{J}_a+\frac{5}{126\cdot 5!}\Gamma^{abcde}[\lambda,J_{abcde}]\,. \qquad (2.3)$$

Apart from F_{ab} and λ^α, the quantities appearing in these equations all arise, as explained in ref. [27], in the θ expansion of J_{abcde}; $\tilde{J}'s$ at first, $K's$ at second, ψ at third, and ω at fourth order in θ. Explicitly, their precise relations to J_{abcde} are given by

$$\begin{aligned}
\tilde{J}_a &= \frac{1}{1680}\Gamma^{bcde}DJ_{bcdea}\,,\\
\tilde{J}_{abc} &= -\frac{1}{12}\Gamma^{de}DJ_{deabc}-\frac{1}{224}\Gamma_{[ab}\Gamma^{defg}DJ_{|defg|c]}\,,\\
\tilde{J}_{abcde} &= DJ_{abcde}+\frac{5}{6}\Gamma_{[ab}\Gamma^{fg}DJ_{|fg|cde]}+\frac{1}{24}\Gamma_{[abcd}\Gamma^{fghi}DJ_{|fghi|e]}\,, && (2.4)\\
K_{ab} &= \frac{1}{5376}(D\Gamma^{cde}D)J_{cdeab}\,,\\
K_{abcd} &= \frac{1}{480}(D\Gamma_{[a}{}^{fg}D)J_{|fg|bcd]}\,, && (2.5)\\
\psi_\alpha &= -\frac{1}{840\cdot 3!\cdot 5!}\Gamma_{abc}{}^{\beta\gamma}\Gamma_{de\,\alpha}{}^{\delta}D_{[\beta}D_\gamma D_{\delta]}J^{abcde}\,, && (2.6)
\end{aligned}$$

and finally

$$w_a=\frac{1}{4032\cdot 4!\cdot 5!}\Gamma^{[\alpha\beta}_{abc}\Gamma^{\gamma\delta]}_{def}D_\alpha D_\beta D_\gamma D_\delta J^{bcdef}\,. \qquad (2.7)$$

Here we have adopted the notation that keeps track of $M^A{}_{BCD}$ and possible structure constants $f^A{}_{BC}$ write them out explicitly: $\lambda\Gamma^{(n)}\lambda$ means $\frac{1}{2}f^A{}_{BC}\lambda^B\Gamma^{(n)}\lambda^C$ and makes sense only for $n=1$ and $n=5$, while $\{\lambda,\tilde{J}_a\}$ instead means $f^A{}_{BC}\lambda^B\tilde{J}^C_a$. In the case of two bosonic quantities, or one fermionic and one bosonic quantity, the anti-commutator indicated by the curly bracket is exchanged for an ordinary commutator bracket as in e.g. $\Gamma^{abcde}[\lambda,J_{abcde}]$. At some places in the formulæ below we use the curly bracket notation just as a way to keep track of structure constants. E.g., the very last term in K_{ab} given below in eq. (2.11) should be read as $-\frac{3}{448}M^A{}_{BCD}f^D{}_{EF}(\lambda^B\Gamma_{ab}{}^i\lambda^C)(\lambda^E\Gamma_i\lambda^F)$.

Before we start analysing the consequences of eq. (2.1), we must make sure that at order θ in J_{abcde},

$$DJ_{abcde}=\tilde{J}_{abcde}+10\Gamma_{[ab}\tilde{J}_{cde]}+5\Gamma_{[abcd}\tilde{J}_{e]}\,, \qquad (2.8)$$

the field in the irreducible representation (00030) vanishes [27]: i.e., that $\tilde{J}_{abcde}=0$, where $\tilde{J}_{abcde}$ is Γ-traceless, must follow as a direct consequence of (2.1). Computing the result of acting with a fermionic covariant derivative on (2.1) and using the lowest order relations $D_\alpha\lambda^\beta=\frac{1}{2}(\Gamma^{ij})_\alpha{}^\beta F_{ij}$ and $D_\alpha F_{ab}=2(\Gamma_{[a}D_{b]}\lambda)_\alpha$, we find trivially that the representation (00030) does not occur in the tensor product of the constituent fields, which implies that the condition is satisfied modulo terms of order α'^4 and higher.

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We can now proceed to derive the lagrangian at order α'^2. The calculation can be simplified as follows. We start from the λ equation above, obtain its explicit form to order α'^2, and derive the action from which it follows. The pure F terms not obtainable this way can be derived by acting with a fermionic derivative on the field equation for λ^α keeping only the pure F^4 terms. Thus we will actually not make use of the A_a equation (2.2) at any point in this paper.

Concentrating on eq. (2.3) we first rewrite it using the following equation also obtained in ref. [27]:

$$\Gamma^{ab}DK_{ab} = -\frac{225}{2}\psi + \frac{5}{2}D^a\tilde{J}_a + \frac{1}{2016}\Gamma^{abcde}[\lambda, J_{abcde}]\,. \tag{2.9}$$

Elimination of ψ gives

$$0 = \not{D}\lambda + \frac{2}{3}D^a\tilde{J}_a + \frac{4}{15}\Gamma^{ab}DK_{ab} + \frac{1}{42\cdot 5!}\Gamma^{abcde}[\lambda, J_{abcde}]\,. \tag{2.10}$$

When acting with spinor derivatives on J_{abcde} to derive the expressions for $\tilde{J}_a$ and K_{ab}, we find that terms of different powers of α' are produced. Higher order terms arise, e.g., by using equations like $\Lambda_{ab} = F_{ab} - \frac{28}{5}K_{ab}$, where Λ_{ab} comes from expanding $D_\alpha\lambda^\beta$ in terms of tensors, $D_\alpha\lambda^\beta = \Lambda\delta_\alpha{}^\beta + \frac{1}{2}(\Gamma^{ij})_\alpha{}^\beta\Lambda_{ij} + \frac{1}{24}(\Gamma^{ijkl})_\alpha{}^\beta\Lambda_{ijkl}$, and the leading term in K_{ab} is of two higher powers of α' than F_{ab}. Dropping all but the leading terms in α' we find:

$$\begin{aligned}
\tilde{J}_a &= -\frac{3}{70}F^{ij}F^{kl}\Gamma_{aijkl}\lambda + \frac{9}{35}F_a{}^iF^{jk}\Gamma_{ijk}\lambda - \frac{1}{5}F_{ij}F^{ij}\Gamma_a\lambda + 2F_{aj}F^{ij}\Gamma_i\lambda + \\
&\quad + \frac{1}{42}(\lambda\Gamma^{ijk}\lambda)\Gamma_{ijk}D_a\lambda - \frac{1}{35}(\lambda\Gamma^{ijk}\lambda)\Gamma_{aij}D_k\lambda + \frac{3}{35}(\lambda\Gamma_a{}^{ij}\lambda)\Gamma_iD_j\lambda\,, \\
K_{ab} &= \frac{4}{7}F_{ij}F^{ij}F_{ab} - \frac{22}{7}F_a{}^iF_b{}^jF_{ij} + \frac{13}{28}F^{ij}(\lambda\Gamma_{abi}D_j\lambda) + \frac{25}{56}F_{ij}(\lambda\Gamma_a{}^{ij}D_b\lambda) + \\
&\quad + \frac{12}{7}F_a{}^i(\lambda\Gamma_bD_i\lambda) + \frac{43}{28}F_a{}^i(\lambda\Gamma_iD_b\lambda) + \frac{3}{28}D_aF^{ij}(\lambda\Gamma_{bij}\lambda) + \\
&\quad + \frac{11}{2688}(\lambda\Gamma^{ijk}\lambda)\{\lambda, \Gamma_{abijk}\lambda\} - \frac{3}{448}(\lambda\Gamma_{ab}{}^i\lambda)\{\lambda, \Gamma_i\lambda\}\,.
\end{aligned} \tag{2.11}$$

In deriving the final form of the equations of motion one needs to rearrange a number of trilinear λ terms in order to have a minimal set of linearly independent terms. We refer to the appendix for the Fierz identities needed. For the complete equation of motion for λ at order α'^2 one finds

$$\begin{aligned}
0 &= \not{D}\lambda + \frac{28}{5}F^{ij}F^{kl}\Gamma_{ijk}D_l\lambda + 24F^i{}_kF^{jk}\Gamma_iD_j\lambda + \\
&\quad - \frac{16}{5}F^{il}D_lF^{jk}\Gamma_{ijk}\lambda + \frac{4}{5}F^{jk}D_iF_{jk}\Gamma^i\lambda - \\
&\quad - \frac{17}{60}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda - \frac{9}{5}(D_i\lambda\Gamma_jD_k\lambda)\Gamma^{ijk}\lambda + \\
&\quad + \frac{1}{5}(\lambda\Gamma^{ijk}\lambda)[F_i{}^l, \Gamma_{jkl}\lambda] + \frac{1}{2}(\lambda\Gamma^{ijk}\lambda)[F_{ij}, \Gamma_k\lambda] + \\
&\quad + \frac{5}{48}F_{ij}\{\lambda, \Gamma^{ijklm}\lambda\}\Gamma_{klm}\lambda - \frac{19}{40}F^{ij}\{\lambda, \Gamma^k\lambda\}\Gamma_{ijk}\lambda + F^{ij}\{\lambda, \Gamma_i\lambda\}\Gamma_j\lambda\,.
\end{aligned} \tag{2.12}$$

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Since this equation is still written in terms of superfields we need to set $\theta = 0$ in order to extract the corresponding component field equation (which is identical to the above equation). Having done so, we now turn to the construction of the action that gives rise to this field equation.

3. The action

To systematise the integration of the equation of motion to a lagrangian, we enumerate the possible terms in the equation of motion,

$$\begin{aligned}
E_1 &= F^{ij}F^{kl}\Gamma_{ijk}D_l\lambda\,, \\
E_2 &= F^i{}_kF^{jk}\Gamma_iD_j\lambda\,, \\
E_3 &= F^{il}D_lF^{jk}\Gamma_{ijk}\lambda\,, \\
E_4 &= F^{jk}D_iF_{jk}\Gamma^i\lambda\,, \\
E_5 &= (D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda\,, \\
E_6 &= (D_i\lambda\Gamma_jD_k\lambda)\Gamma^{ijk}\lambda\,, \\
E_7 &= (\lambda\Gamma^{ijk}\lambda)[F^{lm},\Gamma_{ijklm}\lambda]\,, \\
E_8 &= (\lambda\Gamma^{ijk}\lambda)[F_i{}^l,\Gamma_{jkl}\lambda]\,, \\
E_9 &= (\lambda\Gamma^{ijk}\lambda)[F_{ij},\Gamma_k\lambda]\,, \\
E_{10} &= F_{ij}\{\lambda,\Gamma^{ijklm}\lambda\}\Gamma_{klm}\lambda\,, \\
E_{11} &= F^{ij}\{\lambda,\Gamma^k\lambda\}\Gamma_{ijk}\lambda\,, \\
E_{12} &= F^{ij}\{\lambda,\Gamma_i\lambda\}\Gamma_j\lambda\,,
\end{aligned} \tag{3.1}$$

and write the equation as $0 = \not{D}\lambda + \sum_{n=1}^{12} x_nE_n$, with the numbers x_n given in eq. (2.12), $(x_1,\ldots,x_{12}) = (\frac{28}{5}, 24, -\frac{16}{5}, \frac{4}{5}, -\frac{17}{60}, -\frac{9}{5}, 0, \frac{1}{5}, \frac{1}{2}, \frac{5}{48}, -\frac{19}{40}, 1)$. We also write the lagrangian as

$$\mathcal{L} = \text{pure } F\text{-terms} + \frac{1}{2}\lambda\not{D}\lambda + \sum_{m=1}^{10} a_m\mathcal{L}_m\,, \tag{3.2}$$

where

$$\begin{aligned}
\mathcal{L}_1 &= F^{ij}F^{kl}(\lambda\Gamma_{ijklm}D^m\lambda)\,, \\
\mathcal{L}_2 &= F^i{}_kF^{jk}(\lambda\Gamma_iD_j\lambda)\,, \\
\mathcal{L}_3 &= F^{il}D_lF^{jk}(\lambda\Gamma_{ijk}\lambda)\,, \\
\mathcal{L}_4 &= F^{ij}F_{ij}(\lambda\not{D}\lambda)\,, \\
\mathcal{L}_5 &= F^{ij}D_lF^{kl}(\lambda\Gamma_{ijk}\lambda)\,, \\
\mathcal{L}_6 &= (\lambda\Gamma^{ijk}\lambda)(D_l\lambda\Gamma_{ijk}D^l\lambda)\,, \\
\mathcal{L}_7 &= (\lambda\Gamma^{ijk}\lambda)(D_i\lambda\Gamma_jD_k\lambda)\,,
\end{aligned}$$

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$$\begin{aligned}\mathcal{L}_8 &= (\lambda\Gamma^{ijk}\lambda)(D^l\lambda\Gamma_{lijkm}D^m\lambda)\,,\\ \mathcal{L}_9 &= F^{ij}(\lambda\Gamma_{ijk}\lambda)\{\lambda,\Gamma^k\lambda\}\,,\\ \mathcal{L}_{10} &= F^{ij}(\lambda\Gamma^{klm}\lambda)\{\lambda,\Gamma_{ijklm}\lambda\}\,.\end{aligned} \tag{3.3}$$

The system of equations for determining the a_m's from the x_n's is over-determined, unless two relations hold among the x_n's, namely, $12x_5 - x_6 + 8x_8 = 0$, $x_2 - 192x_{10} - 4x_{12} = 0$. This consistency check is satisfied by the x_n's in eq. (2.12). The solution is

$$(a_1,\ldots,a_{10}) = \left(-\frac{7}{10}, 12, -\frac{8}{5}, -\frac{13}{5}, -\frac{11}{10}, -\frac{1}{30}, -\frac{9}{5}, 0, \frac{11}{20}, \frac{1}{60}\right). \tag{3.4}$$

The only terms in the lagrangian that are not derivable from the equation of motion for λ are the ones containing only F. They are quite easily calculated by taking a spinor derivative on the equation of motion for λ, keeping only pure F-terms, and found to be $-6[\mathrm{tr}\,F^4 - \frac{1}{4}(\mathrm{tr}\,F^2)^2]$, as expected from string theory scattering amplitudes [6] (the traces are over Lorentz indices with F seen as a matrix; the adjoint indices are as usual suppressed). The complete lagrangian at $O(\alpha'^2)$ is thus

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^{ij}F_{ij} + \frac{1}{2}\lambda\not{D}\lambda\\ &+\alpha'^2\Bigg[-6\left(\mathrm{tr}\,F^4 - \frac{1}{4}(\mathrm{tr}\,F^2)^2\right) -\\ &\quad -\frac{7}{10}F^{ij}F^{kl}(\lambda\Gamma_{ijklm}D^m\lambda) + 12F^i{}_kF^{jk}(\lambda\Gamma_iD_j\lambda) -\\ &\quad -\frac{8}{5}F^{il}D_lF^{jk}(\lambda\Gamma_{ijk}\lambda) - \frac{13}{5}F^{ij}F_{ij}(\lambda\not{D}\lambda) - \frac{11}{10}F^{ij}D_lF^{kl}(\lambda\Gamma_{ijk}\lambda) -\\ &\quad -\frac{1}{30}(\lambda\Gamma^{ijk}\lambda)(D_l\lambda\Gamma_{ijk}D^l\lambda) - \frac{9}{5}(\lambda\Gamma^{ijk}\lambda)(D_i\lambda\Gamma_jD_k\lambda) +\\ &\quad +\frac{11}{20}F^{ij}(\lambda\Gamma_{ijk}\lambda)\{\lambda,\Gamma^k\lambda\} + \frac{1}{60}F^{ij}(\lambda\Gamma^{klm}\lambda)\{\lambda,\Gamma_{ijklm}\lambda\}\Bigg] + O(\alpha'^3)\,.\end{aligned} \tag{3.5}$$

Some of the terms, namely those that contain lowest order field equations, may be removed by field redefinitions. One may consider redefinitions of the forms

$$\begin{aligned}\lambda &= \chi + \alpha'^2[\alpha G^{ij}G_{ij}\chi + \beta G^{ij}G^{kl}\Gamma_{ijkl}\chi]\,,\\ A_a &= B_a + \gamma\alpha'^2G^{ij}(\chi\Gamma_{aij}\chi)\,,\end{aligned} \tag{3.6}$$

where G is the field strength of B. The effect of these redefinitions of the physical component fields could also be obtained by redoing the superspace calculation using a new conventional constraint in $\Gamma_a^{\alpha\beta}F_{\alpha\beta}$ corresponding to changing the vector potential, together with a redefinition of the spinor that goes into $\Gamma^{i\,\alpha\beta}F_{i\beta}$. If one wants to continue the calculations to higher order in α', it may be more convenient not to perform them. The redefinitions shift the coefficients a_m as $\delta a_1 = \beta$, $\delta a_3 = -2\beta$,

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$\delta a_4 = \alpha$, $\delta a_5 = -2\beta - \gamma$, $\delta a_9 = \frac{1}{2}\gamma$. By choosing $\alpha = 13/5$, $\beta = 7/10$, $\gamma = -5/2$, we remove the terms $\mathcal{L}_1$, $\mathcal{L}_4$ and $\mathcal{L}_5$, which are the ones containing the lowest order equations of motion, from the lagrangian. We then obtain the lagrangian in its simplest possible form at this level:

$$\begin{aligned}
\mathcal{L}' &= \mathcal{L}'^{(0)} + \alpha'^2 \mathcal{L}'^{(2)} \\
&= -\frac{1}{4}G^{ij}G_{ij} + \frac{1}{2}\chi \not{D}\chi - \\
&\quad -6\alpha'^2 \Big[\operatorname{tr} G^4 - \frac{1}{4}(\operatorname{tr} G^2)^2 - 2G^i{}_k G^{jk}(\chi\Gamma_i D_j \chi) + \frac{1}{2}G^{il}D_l G^{jk}(\chi\Gamma_{ijk}\chi) + \\
&\qquad + \frac{1}{180}(\chi\Gamma^{ijk}\chi)(D_l\chi\Gamma_{ijk}D^l\chi) + \frac{3}{10}(\chi\Gamma^{ijk}\chi)(D_i\chi\Gamma_j D_k\chi) + \\
&\qquad + \frac{7}{60}G^{ij}(\chi\Gamma_{ijk}\chi)\{\chi, \Gamma^k\chi\} - \frac{1}{360}G^{ij}(\chi\Gamma^{klm}\chi)\{\chi, \Gamma_{ijklm}\chi\}\Big] + O(\alpha'^3)\,.
\end{aligned} \tag{3.7}$$

The terms up to quadratic in fermions agree with previous calculations [19], while the quartic fermion terms have not previously been given in the literature. We want to stress the fact that the calculation is exact at this order. It does not in any sense assume that "DF is small", a kind of assumption that is consistent in an abelian theory where it allows a consistent truncation to slowly varying fields, but not in a non-abelian theory, where commutators of covariant derivatives give field strengths. Derivative corrections to the abelian Born-Infeld action are discussed in refs. [8, 12], but for the non-abelian case there seems to be no previous results known at order α'^2 that incorporate all possible derivative terms.

4. Linear and non-linear supersymmetry

Our formalism is manifestly $\mathcal{N} = 1$ supersymmetric, so there is in principle no need to check invariance under linear supersymmetry. However, in this section we will investigate the possibility of having α'-corrected non-linear supersymmetries. The presence of such symmetries would indicate that the non-abelian theory could be viewed as embedded into a theory where all supersymmetries are linearly realised. The appearence of non-abelian parameters in this case would require the introduction of non-abelian θ coordinates in the latter theory which is a delicate enterprise (for a discussion of this and related issues, see ref. [24]). Here, however, the parameter of the non-linear supersymmetry takes values only in the abelian part of the gauge group, so the non-linear supersymmetry found here, although indicating the possibility of embedding the theory into another theory, does not mean that this latter theory should be expressible in terms of non-abelian coordinates.

In order to be able to discuss the relation between the linear and non-linear supersymmetries, we first derive the former. Since linear supersymmetry transformations are defined as translations along the anti-commuting directions of superspace, see e.g. ref. [28], they read, using a parameter ε^α with a tangent space spinor index, $\delta_\varepsilon(\phi)_{\theta=0} = -(\varepsilon^\alpha \hat{D}_\alpha \phi)_{\theta=0}$. Here we use the following conventions: $\hat{D}_\alpha$ is the ordinary flat superspace covariant derivative, without any connection term, i.e., $\{\hat{D}_\alpha, \hat{D}_\beta\} = -T_{\alpha\beta}{}^c \partial_c = -2\Gamma^c_{\alpha\beta}\partial_c$. The gauge-covariant derivatives are $D_a = \partial_a - A_a$, $D_\alpha = \hat{D}_\alpha - A_\alpha$, and commute to $[D_A, D_B\} = -T_{AB}{}^C D_C - F_{AB}$, where the connections and field strengths act in the appropriate representation (in this paper, all fields carry the adjoint representation). A gauge transformation with parameter Λ is denoted $T[\Lambda]$.

Of course, one may freely accompany δ_ε by a gauge transformation. For the multiplet at hand, it is convenient to add a gauge transformation with parameter $\Lambda = \varepsilon^\alpha A_\alpha$, and define $Q_\varepsilon = \delta_\varepsilon + T[\varepsilon^\alpha A_\alpha]$ in order to turn $\hat{D}$ into a gauge-covariant derivative D. This leads to the following simple transformation rules for the component fields ("$\theta = 0$" suppressed):

$$\begin{aligned} Q_\varepsilon \cdot A_a &= \varepsilon^\alpha F_{a\alpha} = (\varepsilon\Gamma_a\lambda) - 7(\varepsilon\tilde{J}_a)\,, \\ Q_\varepsilon \cdot \lambda &= -\varepsilon^\alpha D_\alpha \lambda = \frac{1}{2}\left(F_{ij} - \frac{28}{5}K_{ij}\right)\Gamma^{ij}\varepsilon - \frac{1}{24}(2K_{ijkl} + \frac{7}{30}D^m J_{mijkl})\Gamma^{ijkl}\varepsilon\,. \end{aligned} \tag{4.1}$$

The supersymmetry algebra is ensured by the Bianchi identities. Consider $[Q_\varepsilon, Q_{\varepsilon'}]A_a$. It is obtained as

$$\begin{aligned} Q_\varepsilon \cdot (\varepsilon'^\alpha F_{a\alpha}) - (\varepsilon \leftrightarrow \varepsilon') &= -2\varepsilon^\alpha \varepsilon'^\beta D_{(\alpha}F_{|a|\beta)} \\ &= -\varepsilon^\alpha\varepsilon'^\beta(2\Gamma^i_{\alpha\beta}F_{ia} + D_a F_{\alpha\beta}) \\ &= -2(\varepsilon\Gamma^i\varepsilon')\partial_i A_a + D_a(2(\varepsilon\Gamma^i\varepsilon')A_i - \varepsilon^\alpha\varepsilon'^\beta F_{\alpha\beta})\,, \end{aligned} \tag{4.2}$$

where the dimension 2 Bianchi identity is used in the second step, so that the linear supersymmetries commute to a translation and a gauge transformation for any $F_{\alpha\beta}$,

$$[Q_\varepsilon, Q_{\varepsilon'}] = -2(\varepsilon\Gamma^i\varepsilon')\partial_i + T[2(\varepsilon\Gamma^i\varepsilon')A_i - \varepsilon^\alpha\varepsilon'^\beta F_{\alpha\beta}]\,. \tag{4.3}$$

On any tensor, the same algebra may of course be derived directly from the algebra of $Q_\varepsilon = -\varepsilon^\alpha \hat{D}_\alpha + T[\varepsilon^\alpha A_\alpha] = -\varepsilon^\alpha D_\alpha$.

For gauge groups containing a U(1) factor, the undeformed action has a second, non-linearly realised supersymmetry:

$$\left.\begin{aligned} \delta_\eta A_a &= 0 \\ \delta_\eta \lambda &= \eta \end{aligned}\right\} \quad \Longrightarrow \quad [\delta_\eta, \delta_{\eta'}] = 0\,, \tag{4.4}$$

with η taking values in the $\mathfrak{u}(1)$ subalgebra. We want to examine what happens to this symmetry when higher order corrections are turned on. Let us denote the above undeformed transformation $\delta^{(0)}_\eta$. We then expect that δ_η, if it remains unbroken by the α'^2-corrections, receives modifications $\alpha'^2\delta^{(2)}_\eta$. Invariance means that $\delta^{(0)}_\eta \mathcal{L}'^{(2)}$ must be canceled against

$$\delta^{(2)}_\eta \mathcal{L}'^{(0)} = \left(D_j G^{ji} + \frac{1}{2}\{\chi, \Gamma^i\chi\}\right)\delta^{(2)}_\eta B_i + \delta^{(2)}_\eta \chi \not{D}\chi\,. \tag{4.5}$$

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We therefore calculate $\delta^{(0)}_\eta \mathcal{L}'^{(2)}$, and find that it is of the form (4.5) modulo total derivatives This property is highly non-trivial, and relies on the precise numerical coefficients in the action at order α'^2 (it requires three linear relations between the six coefficients of terms in $\mathcal{L}'$ containing χ). The modified transformations[4] are read off by comparing $\delta^{(0)}_\eta \mathcal{L}'^{(2)}$ to eq. (4.5):

$$\begin{aligned}
\delta_\eta B_a &= -6\alpha'^2\Big[2G_{ai}(\eta\Gamma^i\chi) - G^{ij}(\eta\Gamma_{aij}\chi)\Big]\,,\\
\delta_\eta\chi &= \eta - 6\alpha'^2\bigg[\frac{1}{2}G^{ij}G_{ij}\eta + \frac{1}{4}G^{ij}G^{kl}\Gamma_{ijkl}\eta - 2(\eta\Gamma^i\chi)D_i\chi + \frac{1}{15}(\eta\Gamma^{ijk}\chi)\Gamma_{ij}D_k\chi +\\
&\qquad + \frac{19}{25}(\eta\Gamma^i\chi)\Gamma_i\not{D}\chi - \frac{1}{90}(\eta\Gamma^{ijk}\chi)\Gamma_{ijk}\not{D}\chi\bigg]\,,
\end{aligned}\tag{4.6}$$

or, in terms of the original variables A and λ:

$$\begin{aligned}
\delta_\eta A_a &= -6\alpha'^2\Big[2F_{ai}(\eta\Gamma^i\lambda) - \frac{1}{6}F^{ij}(\eta\Gamma_{aij}\lambda)\Big]\,,\\
\delta_\eta\lambda &= \eta - 6\alpha'^2\bigg[\frac{1}{15}F^{ij}F_{ij}\eta + \frac{2}{15}F^{ij}F^{kl}\Gamma_{ijkl}\eta - 2(\eta\Gamma^i\lambda)D_i\lambda + \frac{1}{15}(\eta\Gamma^{ijk}\lambda)\Gamma_{ij}D_k\lambda +\\
&\qquad + \frac{19}{25}(\eta\Gamma^i\lambda)\Gamma_i\not{D}\lambda - \frac{1}{90}(\eta\Gamma^{ijk}\lambda)\Gamma_{ijk}\not{D}\lambda\bigg]\,.
\end{aligned}\tag{4.7}$$

The commutator of two non-linear supersymmetry transformations is read off directly from the transformations (4.6) or (4.7):

$$\begin{aligned}
[\delta_\eta,\delta_{\eta'}]A_a &= -24\alpha'^2(\eta\Gamma^i\eta')F_{ia}\,,\\
[\delta_\eta,\delta_{\eta'}]\lambda &= -24\alpha'^2(\eta\Gamma^i\eta')D_i\lambda\,,
\end{aligned}\tag{4.8}$$

modulo field equations. Remember that the coeffients M_{ABCD} are still present and suppressed. Redefining the transformation parameter as $\varrho = 2\sqrt{3\mu}\,\alpha'\eta$, so that ϱ, like ε, has mass dimension $-1/2$, gives a standard supersymmetry algebra:

$$[S_\varrho, S_{\varrho'}] = -2(\varrho\Gamma^i\varrho')\partial_i + T[2(\varrho\Gamma^i\varrho')A_i]\,,\tag{4.9}$$

provided that when we decompose the adjoint indices as $A=(0,A')$, 0 denoting the U(1) factor, M_{ABCD} satisfies $M_{0000}=\mu$, $M_{00A'B'}=\mu\delta_{A'B'}$. A straightforward but non-trivial calculation also shows that the commutator between a linear supersymmetry (from eq. (4.1)) and a non-linear one yields a gauge transformation,

$$[Q_\varepsilon, S_\varrho] = T\left[-\frac{1}{2\sqrt{3\mu}\alpha'}(\varepsilon\Gamma_i\varrho)x^i + \frac{\alpha'}{6\sqrt{3\mu}}(\varepsilon\Gamma^{ijk}\varrho)(\lambda\Gamma_{ijk}\lambda)\right]\,,\tag{4.10}$$

modulo field equations (we have satisfied ourselves with doing the calculation act-

[4]The terms in the transformations containing $\not{D}\chi$ or $\not{D}\lambda$ are only relevant for the invariance of the action, not for the supersymmetry algebra, which only closes on shell. The linear supersymmetry transformations do not get corrected by such terms since they are derived from the superspace formulation, and thus are on-shell transformations.

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ing on A_a). The shorthand notation of this equation is the same as in the action: quadratic expressions are contracted with δ_{AB}, so the first term is an U(1) gauge transformation, quartic ones with M_{ABCD}, so the second one contains M_{0ABC}. Unlike the ordinary (undeformed) super–Yang-Mills, where the non-linear supersymmetry is abelian, the α'-corrected theory may be seen as a theory with partially broken $\mathcal{N} = (2,0)$ chiral supersymmetry where λ is the Goldstone fermion. Starting from the $\mathcal{N} = (2,0)$ supersymmetry algebra and reintroducing the α' rescaling of the S generators, $\eta \sim \alpha'^{-1}\varrho$, we can understand the supersymmetry of the undeformed super–Yang-Mills theory as a contraction as $\alpha' \to 0$ of the $\mathcal{N} = (2,0)$ supersymmetry algebra.

5. The structure of the order α'^2 interactions

The tensor M_{ABCD} is totally symmetric in the four adjoint indices. Table 1 (last column) gives the number P_4 of linearly independent invariant tensors of this kind for simple algebras. For all algebras, there is always the tensor $\delta_{(AB}\delta_{CD)}$, and in the cases where the number is 1, this is the only possible form of M. Algebras of $\mathfrak{su}(N)$ type, $N \geq 4$, have in addition the tensor $d_{(AB}{}^{E}d_{CD)E}$, and for $\mathfrak{sp}$ and $\mathfrak{so}$ algebras there is a quartic invariant that can not be expressed in terms of a d-symbol with three indices (it can be taken as $\mathrm{Tr}_f(T_{(A}T_BT_CT_{D)})$). $\mathfrak{so}(8)$ has one more quartic invariant corresponding to the pfaffian. Linear supersymmetry does not restrict M further, so in the cases where $P_4 > 1$ the coefficients of the different possible invariants are unrelated.

Lie Algebra $\mathfrak{g}$	P_3	P_4
A_1	-	1
A_2	1	1
A_n, $n \geq 3$	1	2
B_n, $n \geq 2$	-	2
C_n, $n \geq 3$	-	2
D_4	-	3
D_n, $n \geq 5$	-	2
exceptional	-	1

Table 1: The number of singlets in the totally symmetric product of 3 and 4 adjoints (could be enlarged to include higher Casimirs).

If the algebra is not simple, more possibilities arise. Of particular interest is the situation when $\mathfrak{g} = \mathfrak{g}' \oplus \mathfrak{u}(1)$, which is the case for multiple branes, $\mathfrak{g} = \mathfrak{u}(N) \simeq \mathfrak{su}(N) \oplus \mathfrak{u}(1)$. Then, we decompose the adjoint index as $A = (0, A')$, 0 denoting the $\mathfrak{u}(1)$ part, and the possibilities are

$$\begin{aligned} M_{A'B'C'D'} &= a\, d_{(A'B'}{}^{E'}d_{C'D')E'} + b\,\delta_{(A'B'}\delta_{C'D')}\,, \\ M_{0A'B'C'} &= c\, d_{A'B'C'}\,, \\ M_{00A'B'} &= d\,\delta_{A'B'}\,, \\ M_{000A'} &= 0\,, \\ M_{0000} &= \mu\,. \end{aligned} \tag{5.1}$$

Any values of a, b, c, d, μ are consistent with linear supersymmetry. If one in addition demands the second non-linearly realised supersymmetry, one has to, as we saw in the previous section, take $d = \mu \neq 0$ in order for these supersymmetries to commute to a translation. The remaining constants are unspecified. The "symmetric

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trace" prescription of Tseytlin [14], used without referring to string theory, becomes identical to our $M_{ABCD} \propto \mathrm{Tr}(T_{(A}T_BT_CT_{D)})$. However, in string theory the trace is by necessity in the fundamental representation which implies that, for $\mathfrak{su}(N)$, the generators satisfy

$$T_{A'}T_{B'} = \frac{1}{N}\delta_{A'B'} + \frac{1}{2}(d_{A'B'}{}^{C'} + f_{A'B'}{}^{C'})T_{C'}\,, \tag{5.2}$$

and hence M_{ABCD} contains a specific combination of all terms in eq. (5.1): $a = 1$, $b = 4/N$, $c = 2/\sqrt{N}$, $d = \mu = 4/N$. These restrictions are consistent with the condition we obtained from non-linear supersymmetry namely $\mu = d$.

6. Conclusions and comments

In this paper we have derived the α'^2-corrections (linear supersymmetry does not allow any corrections at order α' [27]) to the supersymmetric Yang-Mills theory in ten dimensions. The corrections obtained are the most general ones compatible with linear supersymmetry which is implemented by the use of superspace. As shown in previous work [29, 27] the superspace Bianchi identities can be solved in full generality in terms of the component fields of a self-dual five-form superfield J_{abcde}. By expressing this superfield in terms of the physical fields, F_{ab}, λ^α, and covariant derivatives, D_a, we derived in sections 2 and 3 the action resulting from the α'^2 terms in J_{abcde} not removable by field redefinitions of the superfield A_α. The action includes quartic fermion terms not previously given in the literature as well as a precise account of the freedom left after imposing (linear) supersymmetry.

This remaining freedom is given by $M^A{}_{BCD}$ which is symmetric in all four adjoint group indices and should be constructed from the invariant group tensors, which in the $\mathfrak{su}$ case are δ_{AB}, d_{ABC} and f_{ABC}. This is elaborated upon in section 5. Each independent way of writing $M^A{}_{BCD}$ corresponds to a new supersymmetric invariant that most likely will be given by an infinite power series in α'. This phenomenon will probably repeat itself at higher orders producing leading terms in an infinite set of new invariants, but it should be stressed that it can not be excluded that no additional freedom (i.e., new invariants) arises at higher orders. It is also important to note that the non-linearities of the theory will in general cause the superinvariants to mix at non-leading orders.

However, the derivation of the action order by order in α' is a rather lengthy iterative process that contains some interesting aspects that could change this picture. One such feature of the solution to the superspace Bianchi identities obtained in [27] is that it requires the the tensor field in the irreducible representation (00030) of $Spin(1,9)$ at first order in the θ expansion of J_{abcde} to vanish. This is the only non-trivial test of a non-vanishing $F_{\alpha\beta}$ defining the theory. This starts to affect the analysis at order α'^3 where one has to make sure that the terms that are allowed by

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group theory actually vanish. At order α'^4, one has in addition to prove that the terms that do not vanish are exactly the ones that cancel the α'^4 terms found in the analysis of the (00030) condition in this paper (see comments in section 2). This analysis may turn out to provide rather severe restrictions at higher orders. Also, as we will discuss in a forthcoming publication [31], although there is a rather large number of possible terms contributing to J_{abcde} at order α'^3 most of them seem to be removable by field redefinitions of A_α similar to the ones utilised in this paper at order α'^2. This amounts to identifying elements in the spinorial cohomology discussed in refs. [27, 32] when expressed in terms of fields in the vector multiplet.

Although the Born-Infeld action for a constant and abelian field strength does not contain any terms of odd powers of α', such terms will most likely appear as soon as either of these restrictions is lifted. Derivative corrections to the abelian theory have been discussed by several authors, see, e.g., refs. [8, 11, 12] showing that α'^3 terms actually do not arise. In the non-abelian case, the situation is unclear even for the α'^3 order terms (an early superstring computation of such terms can be found in ref. [33]). Applying our methods to this case will hopefully clarify the situation.

Another intriguing feature of the corrections obtained here is that they allow for a second non-linearly realised supersymmetry provided, as explained in section 4, only a minor restriction is imposed on $M^A{}_{BCD}$. This restriction is compatible with the one obtained from string theory. It would be very interesting to investigate how this non-linearly realised supersymmetry generalises to higher orders, and to see if it can be understood as resulting from embedding the theory in a similar theory where all supersymmetries are linearly realised along the lines of ref. [15]. For non-abelian theories such embeddings have not yet been studied in any detail. However, some results that might be related to non-linearly realised symmetries with non-abelian valued parameters can be found in refs. [22, 23, 24]. What our work explains is how a non-linear supersymmetry with an abelian parameter can be made to act on non-abelian fields in a non-trivial manner extending the linear symmetry to an $\mathcal{N}=2$ supersymmetry. It is clear that from the perspective of $\mathcal{N}=2$ supersymmetry, the corrections at order α'^2 considered in the present paper are special — it is exactly this modification that changes the algebra from a "trivial" abelian shift in the spinor to an ordinary supersymmetry algebra, indicating that the theory may be embedded in a theory with linear $\mathcal{N}=2$ supersymmetry with the $\mathfrak{u}(1)$ part of λ as the Goldstone fermion. We find it very striking that the requirement of non-linear supersymmetry contains practically no information on the structure of the interactions not already implied by linear supersymmetry. It will be interesting to examine whether this statement continues to hold at higher orders.

Acknowledgments

This work is partly supported by EU contract HPRN-CT-2000-00122 and by the

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Swedish Natural Science Research Council.

A. Spinors and Fierz identities

We use real chiral (Majorana-Weyl) spinors throughout the paper. The two chiralities are distinguished by subscript for the (00010) representation (e.g. D_α) and superscript for the (00001) (e.g. λ^α). The Γ-matrices are thus not Dirac matrices but "Pauli matrices"; we use the same notation Γ for $\Gamma^a_{\alpha\beta}$ and $\Gamma^{a\,\alpha\beta}$.

In deriving the final form of the equations of motion one needs to rearrange a number of trilinear λ terms. For this purpose the following Fierz identities are quite useful. The general identity

$$(\lambda A\{\lambda), B\lambda\} = \frac{1}{16}\{\lambda,\Gamma^i\lambda\}B\Gamma_i A^t\lambda + \frac{1}{32\cdot 5!}\{\lambda,\Gamma^{ijklm}\lambda\}B\Gamma_{ijklm}A^t\lambda \tag{A.1}$$

leads to the various Fierz identities

$$\begin{aligned}
(\lambda\Gamma_{[a}\{\lambda),\Gamma_{b]}\lambda\} &= \frac{1}{16}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda + \frac{1}{96}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda\,,\\
(\lambda\Gamma^i\{\lambda),\Gamma_{abi}\lambda\} &= \{\lambda,\Gamma_{[a}\lambda\}\Gamma_{b]}\lambda - \frac{3}{8}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda + \frac{1}{48}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda\,,\\
(\lambda\Gamma_{abi}\{\lambda),\Gamma^i\lambda\} &= \{\lambda,\Gamma_{[a}\lambda\}\Gamma_{b]}\lambda + \frac{3}{8}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda - \frac{1}{48}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda\,,\\
(\lambda\Gamma_{ij[a}\{\lambda),\Gamma^{ij}{}_{b]}\lambda\} &= \frac{7}{4}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda - \frac{1}{24}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda\,,\\
(\lambda\Gamma^{ijk}\{\lambda),\Gamma_{abijk}\lambda\} &= 42\{\lambda,\Gamma_{[a}\lambda\}\Gamma_{b]}\lambda - \frac{21}{4}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda - \frac{3}{8}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda\,,\\
(\lambda\Gamma_{abijk}\{\lambda),\Gamma^{ijk}\lambda\} &= 42\{\lambda,\Gamma_{[a}\lambda\}\Gamma_{b]}\lambda + \frac{21}{4}\{\lambda,\Gamma^i\lambda\}\Gamma_{abi}\lambda + \frac{3}{8}\{\lambda,\Gamma_{abijk}\lambda\}\Gamma^{ijk}\lambda\,.
\end{aligned} \tag{A.2}$$

Eq. (A.1) is most easily derived by recalling that $(\lambda A\{\lambda),B\lambda\}$ means $(\lambda^A A\lambda^B)(B\lambda^C)_\alpha - (\lambda^A A\lambda^C)(B\lambda^B)_\alpha$ (the B and C indices are contracted by a structure constant, but it is only the symmetry that is relevant here), and then use the following expansion of the product of two spinors anti-symmetrised in the adjoint indices and hence symmetrised in the spinor indices:

$$\lambda^{[B}_\alpha\lambda^{C]}_\beta = \frac{1}{16}(\Gamma^a)_{\alpha\beta}(\lambda^B\Gamma_a\lambda^C) + \frac{1}{32\cdot 5!}(\Gamma^{abcde})_{\alpha\beta}(\lambda^B\Gamma_{abcde}\lambda^C)\,, \tag{A.3}$$

where the coefficients follow directly from the trace formulæ for chirally projected 16 by 16 Γ-matrices:

$$(\Gamma^a)^{\alpha\beta}(\Gamma^b)_{\alpha\beta} = 16\eta^{ab}\,, \tag{A.4}$$

and

$$(\Gamma^{a_1\ldots a_5})^{\alpha\beta}(\Gamma_{b_1\ldots b_5})_{\alpha\beta} = \mathrm{tr}\left(\frac{1}{2}(\hat{1}+\hat{\Gamma}^{11})\hat{\Gamma}^{a_1\ldots a_5}\hat{\Gamma}^{b_1\ldots b_5}\right)$$

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$$
\begin{aligned}
&= 16\cdot 5!\delta^{a_1\dots a_5}_{b_1\dots b_5} + 16\epsilon^{a_1\dots a_5}{}_{b_1\dots b_5}\,,\\
(\Gamma^{a_1\dots a_5})_{\alpha\beta}(\Gamma_{b_1\dots b_5})^{\alpha\beta} &= \mathrm{tr}\left(\frac{1}{2}(\hat{1}-\hat{\Gamma}^{11})\hat{\Gamma}^{a_1\dots a_5}\hat{\Gamma}^{b_1\dots b_5}\right)\\
&= 16\cdot 5!\delta^{a_1\dots a_5}_{b_1\dots b_5} - 16\epsilon^{a_1\dots a_5}{}_{b_1\dots b_5}\,,
\end{aligned} \tag{A.5}
$$

where tr refers to traces over non-chiral 32 by 32 $\hat{\Gamma}$ matrices.

Other useful Fierzes, involving expressions like $(\lambda\Gamma^i D^j\lambda)\Gamma_i D_j\lambda$ which with an explicit M becomes $M^A{}_{BCD}(\lambda^B\Gamma^i D^j\lambda^C)\Gamma_i D_j\lambda^D$, are:

$$
\begin{aligned}
(\lambda\Gamma^i D^j\lambda)\Gamma_i D_j\lambda &= \frac{1}{24}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda\,,\\
(\lambda\Gamma^i D^j\lambda)\Gamma_j D_i\lambda &= \frac{1}{48}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda + \frac{1}{4}(D_i\lambda\Gamma_j D_k\lambda)\Gamma^{ijk}\lambda\,,\\
(\lambda\Gamma^{ijk} D^l\lambda)\Gamma_{ijk} D_l\lambda &= -\frac{1}{2}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda\,,\\
(\lambda\Gamma^{ijk} D^l\lambda)\Gamma_{ijl} D_k\lambda &= -\frac{1}{24}(D^l\lambda\Gamma_{ijk}D_l\lambda)\Gamma^{ijk}\lambda + \frac{5}{2}(D_i\lambda\Gamma_j D_k\lambda)\Gamma^{ijk}\lambda\,,
\end{aligned} \tag{A.6}
$$

where we have used also the lowest order field equation $\not{D}\lambda = 0$.

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